$$R_W = R_{f,d}(h_f/h_0)^{1/2}\,\delta U[b(x)/d] \tag{6}$$

Using the results of linearized axisymmetric wake theory as given, for example, by Goldstein,[5]

$$\delta U \sim \frac{C_{Df}R_{f,d}}{x/d} \qquad \frac{b(x)}{d} \sim \left(\frac{x/d}{R_{f,d}}\right)^{1/2} \tag{7}\ddagger$$

where C_{Df} is the wake drag coefficient. Applying the Crocco integral ($P_r = 1$, $dp/dx \approx 0$)

$$\frac{h_0}{h_f} \approx 1 + \frac{(2\,\beta_1^2 - 1 + \alpha)\,M_\infty^2/5}{1 + M_\infty^2/5\,(1 - \beta_1^2)}\,\delta U \tag{8}$$

for $\delta U < 1$ when $M_\infty^2/5$ is large. Thus

$$h_0/h_f \approx 0.3\,M_\infty^2\,\delta U \text{ (sharp bodies)}$$
$$h_0/h_f \approx 3\,\delta U \text{ (blunt bodies)} \tag{9}$$

Inserting (7) and (9) into (6) gives

$$R_W \sim \frac{1.8(C_{Df})^{1/2}}{M_\infty}\,R_{f,d} \text{ (sharp)}$$
$$R_W \sim 0.6(C_{Df})^{1/2}\,R_{f,d} \text{ (blunt)} \tag{10}$$

From the analysis of Ref. 2, $C_{Df} \sim R_{\infty,d}^{-1/2}$; hence (10) indicates that, over the same approximate range of $R_{\infty,d}$, equal values of R_W are obtained for sharp and blunt bodies when

$$(3R_{f,d}/M_\infty)_{\text{sharp}} \approx (R_{f,d})_{\text{blunt}} \tag{11}$$

Dependence on Flight Speed

If isentropic flow behind the normal bow shock of a blunt body is assumed, then for a given freestream temperature and a constant γ of $\frac{7}{5}$,

$$R_{f,d} \sim \frac{M_\infty^8[5 + M_\infty^2(1 - \beta_1^2)]^2}{(7\,M_\infty^2 - 1)^{5/2}\,(5 + M_\infty^2)^{7/2}}\,p_\infty d$$

Hence, for $M_\infty \gg 1$,

$$R_{f,d} \sim p_\infty d \text{ (blunt)} \tag{12}$$

alone, and for this reason the flight speed is irrelevant so long as it is sufficiently high. Experimental data obtained thus far have been at flight speeds insufficient for this conclusion to be strictly valid; however, in these cases the dependence on flight speed should be weak. But for sharp bodies

$$R_{f,d} \sim M_\infty p_\infty d \text{ (sharp)} \tag{13}$$

and the flight speed enters directly into this Reynolds number.

For the stability Reynolds number, inserting (12) or (13) into (10) gives

$$R_W \sim 1.8(C_{Df})^{1/2}\,p_\infty d \text{ (sharp)}$$
$$R_W \sim 0.6(C_{Df})^{1/2}\,p_\infty d \text{ (blunt)} \tag{14}$$

indicating that, so long as $C_{Df}^{1/2} \sim (R_{\infty,d})^{-1/4}$ is about constant, stability of both sharp and blunt-body wakes depends on $p_\infty d$ alone; moreover, the lower critical value of $p_\infty d$ for blunt bodies may be somewhat larger than the value for sharp bodies.

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‡ The same results for (7) as well as (8) also may be obtained by applying the analysis of Kubota[6] for a compressible wake.

Solid Propellant Exhaust Simulation

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THE purpose of this paper is to record a procedure for simulating exhausts of solid propellant engines by means of liquid, gaseous, and slurry systems. The method was devised some years ago and since has been developed and demonstrated, originally at Aeronutronic[1] and later at several other installations. The concept has many obvious applications and a major cost advantage over testing exhaust effects by use of solid rocket grains.

A given solid propellant can, in principle, be duplicated identically by any of a large number of combinations of liquid and/or gaseous or slurry systems. The duplications can be achieved with regard to both chamber temperature and chemical composition.

The composition and thermodynamic properties of an exhaust stream in thermodynamic equilibrium are determined uniquely by the atomic composition, the temperature, and the pressure. The pressure is fixed by the engine design, and the temperature is governed by the enthalpy of formation of the propellant. Consequently, in order to simulate the exhaust of a given solid propellant engine with a liquid, gaseous, or slurry system, the important parameters to consider are atomic composition and enthalpy of formation. The procedure is best illustrated by means of an example.

Suppose it is required to simulate the exhaust from an aluminized solid propellant of composition $C_a\,H_b\,O_c\,N_d\,Cl_eAl_f$ and enthalpy of formation $Hf/100$ g. If one selects seven chemicals, which among them contain the desired elements, one can, by a simple mass- and heat-balance process, duplicate the composition and enthalpy of formation precisely. In general, $N + 1$ ingredients are needed where N is the number of elements in the solid propellant. In the case cited, the components of the mixture might be chosen as $Al(s)$, $C_2HCl_3(l)$ (trichloroethylene), $C_2H_8N_2$ (l) (unsymmetrical dimethylhydrazine), $N_2(g)$, $O_2(g)$, $CH_{1.942}(l)$ (JP-4), and $H_2O(l)$. The aluminum would be used as a slurry with one or more of the liquids. The following equations then are set up:

$$a = 2\,N_{C_2H_3Cl} + 2\,N_{C_2H_8N_2} + N_{CH_{1.942}}$$
$$b = N_{C_2HCl_3} + 8\,N_{C_2H_8N_2} + 1.942\,N_{CH_{1.942}} + 2\,N_{H_2O}$$
$$c = 2\,N_{O_2} + N_{H_2O}$$
$$d = 2\,N_{C_2H_8N_2} + 2N_{N_2}$$

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[1] Carver, G. P. et al. (in preparation).

$$e = 3N_{C_2HCl_3}$$

$$f = N_{Al}$$

$$Hf = \sum_i N_i Hf_i$$

where

$N_i \equiv$ moles of component i/100 g of propellant
$Hf_i \equiv$ molar enthalpy of formation of component i

Given the enthalpies of formation of the components, the equations may be solved to find the composition of a mixture which will provide an exhaust indistinguishable from that of the solid propellant. The simulation will be valid only if equilibrium combustion occurs in both cases. A further limitation is that in the case of multiphase exhausts the particle size distribution will not, necessarily, be duplicated.

In practice, a proposed composition may for some reason prove undesirable. In the case cited, for example, the slurry required might be too concentrated for practical use, or there might be ignition problems. In such an event, a liquid aluminum derivative might be used or the gaseous oxygen might be replaced by a liquid oxidizer. The only requirement is that an element and enthalpy balance be obtained, and the limitation is thus only with regard to the number of suitable compounds of known composition and enthalpy of formation which are available.

Exact simulation may not always be necessary or practical. Formulation might be greatly simplified by omission of one of the minor components, with little effect on the exhaust characteristics. Also, systematic variation of the exhaust temperature and chemistry might be desired. In the foregoing example, for instance, the component $C_2H_8N_2(l)$ and the heat balance equation might be omitted in the calculation. The nitrogenous compounds in the exhaust would then arise entirely from $N_2(g)$. The atomic composition of the exhaust would be the same as before, but the temperature and chemical composition would be different.

Thermoelastic Vibrations of Heterogeneous Membranes and Inextensional Plates

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IN Ref. 1 a general nonlinear thermoelastic theory was established for thin heterogeneous aeolotropic plates. In the following, the author specializes the general equations to the cases of 1) membranes, 2) pseudo-membranes, and 3) inextensional plates, including thermally induced transverse vibrations.

The two simultaneous nonlinear equations for w and F established previously[1] are

$$L_1 w - L_3 F = p + L_B N + L_t M + K(F,w) \quad (1)$$

$$L_2 F + L_3 w = L_A N - \tfrac{1}{2}K(w,w) \quad (2)$$

where

$$L_A N = L_{Ax} N_{xt} + L_{Ay} N_{yt} + L_{As} N_{st} \quad (3)$$

$$L_B N = L_{Bx} N_{xt} + L_{By} N_{yt} + L_{Bs} N_{st} \quad (4)$$

$$L_t M = M_{xt,xx} + 2M_{st,xy} + M_{yt,yy} \quad (5)$$

$$K(F,w) = F_{,yy} w_{,xx} - 2F_{,xy} w_{,xy} + F_{,xx} w_{,yy} \quad (6)$$

$$\tfrac{1}{2}K(w,w) = w_{,xx} w_{,yy} - w_{,xy}^2 \quad (7)$$

and the other operators are given in Ref. 1.

Note that the linear coupling operator L_3 vanishes for homogeneous plates and for some special classes of heterogeneity,[2] e.g., for symmetrically laminated plates. For such systems a membrane state exists whenever $L_1 \to 0$, and an inextensional state exists whenever $L_2 \to 0$. However, for a general heterogeneous system characterized by Eqs. (1) and (2), one distinguishes between a pseudo-membrane state where $L_1 = 0$ and a classical membrane state where, in addition to L_1, the operator L_3 also vanishes.

The finite deflection equations of a heterogeneous aeolotropic pseudo-membrane, which also undergoes thermal gradients, are as follows:

$$-L_3 F = p + L_B N + L_t M + K(F,w) \quad (8)$$

$$L_2 F + L_3 w = L_A N - \tfrac{1}{2}K(w,w) \quad (9)$$

It is noted that the pseudo-membrane state is not free of bending moments, as can be seen from Eqs. (29) of Ref. 1. One also may distinguish between a pseudo-inextensional state, where $L_2 = 0$ in Eqs. (1) and (2), and a classical inextensional state, where L_3, as well as L_2, vanish.

The equations of a pseudo-inextensional heterogeneous plate take the form

$$L_1 w - L_3 F = p + L_B N + L_t M + K(F,w) \quad (10)$$

$$L_3 w = L_A N - \tfrac{1}{2}K(w,w) \quad (11)$$

Note that the assumption of pseudo-inextensibility reduces the coupled eighth-order system of simultaneous Eqs. (1) and (2) to two successive fourth-order equations. The coupling of w and F will, however, still be retained in the boundary conditions, as can be seen from Eqs. (10) and (19) of Ref. 2.

So far the effect of inertia was disregarded in the analyses; however, in problems of thermally induced vibrations of plates, it must be considered. To account for transverse vibrations of the heterogeneous systems, apply Eqs. (8) and (9) or (10) and (11) as the case is, except that the lateral load p is replaced by an inertia load term of the form $-R_0 w_{,tt}$, where

$$R_0 = \int_0^h \rho \, dz \quad (12)$$

and $\rho = \rho(z)$ is the density of the plate material.

Accordingly, the differential equations for free pseudo-membrane lateral vibrations or free inextensional transverse vibrations are Eqs. (8) and (9) or Eqs. (10) and (11), respectively, where now

$$w = w(x,y,t) \quad (13)$$

$$F = F(x,y,t) \quad (14)$$

and neglecting longitudinal inertia

$$p = p(x,y,t) = -R_0 w_{,tt} \quad (15)$$

Equations (8) and (9) include, as a special case, Föppl's[3] equations for finite deflections of homogeneous membranes.

Equations (10) and (11) with (13–15) include, as a special case, Boley and Barber's[4] equation for thermally induced vibrations of homogeneous plates.

The author finally mentions a particular case of deflection of a pseudo-membrane to a cylindrical surface whose axis is parallel to the y axis.

Considering uniform heating, Eqs. (8) and (9) are simplified to

$$B_{yx} * F_{,xxxx} = p + F_{,yy} w_{,xx} \quad (16)$$

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